2021
B.A./B.Sc. (General), Fifth Semester
Mathematics
Paper – II: Modern Algebra

Time allowed: 3 Hours

Max. Marks: 30

NOTE: Attempt _five_ questions in all, selecting atleast two questions from each Unit.

x-x-x

UNIT – I

I. a) If a and *b* be any two elements of group *G* such that $ab == ba$ and $(0(a), (0(b)) = 1$, then show that $O(ab) = O(a)O(b)$.

b) Find out order of elements of Q_8, quarternions group. Is Q_8 abelian? Justify. (3,3)

II. a) State and prove Lagrange's theorem. Is converse true?

b) Let H be a subgroup of G Show that $O(H) = O(x^{-1} Hx), \forall x \in G.$ (3, 3)

III. a) Let K be a normal subgroup of a group G and L be any subgroup of G, then prove that

$$LK/K \cong L/{L \cap K}$$

b) Find all subgroups of $Z/{21Z}$ (4,2)

IV. a) Prove that the subset A_n consisting of all the even permutations in S_n is a normal subgroup of index 2.

b) Let $p = (1\ 2\ 3)$ and $q = (1\ 3\ 2)$ be two permutation on the set $\{1,2, 3\}$. Calculate $pqp^{-1}.$ (4, 2)

UNIT – II

V. a) Prove that a commutative ring with unity is a field iff it does not have any proper ideal.

b) Show that $< 4 >$, ideal generated by 4 is maximal ideal in ring 2Z of even integers. (4,2)

P.T.O.

VI. a) Show that a ring R is commutative iff $(a + b)^2 = a^2 + b^2 + 2ab$ for all $a, b \in R$.

b) Find the field of quotients of the integral domain $Z[\sqrt{2}]$. (3, 3)

VII. a) Prove that every field is integral domain. Is converse true? Justify.

b) Let $f : R \to R'$ be an onto homomorphism function. Then prove that

$$\frac{R}{Kerf} \cong R'$$ (3,3)

VIII. a) If R is an integral domain then prove that R[x], the ring of polynomials over R is also an integral domain.

b) Let R be an integral domain and f(x), $g(x) \in R[x]$. Prove that

deg (f(x).g(x))=deg(f(x))+deg(g(x)). (3,3)

x-x-x

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